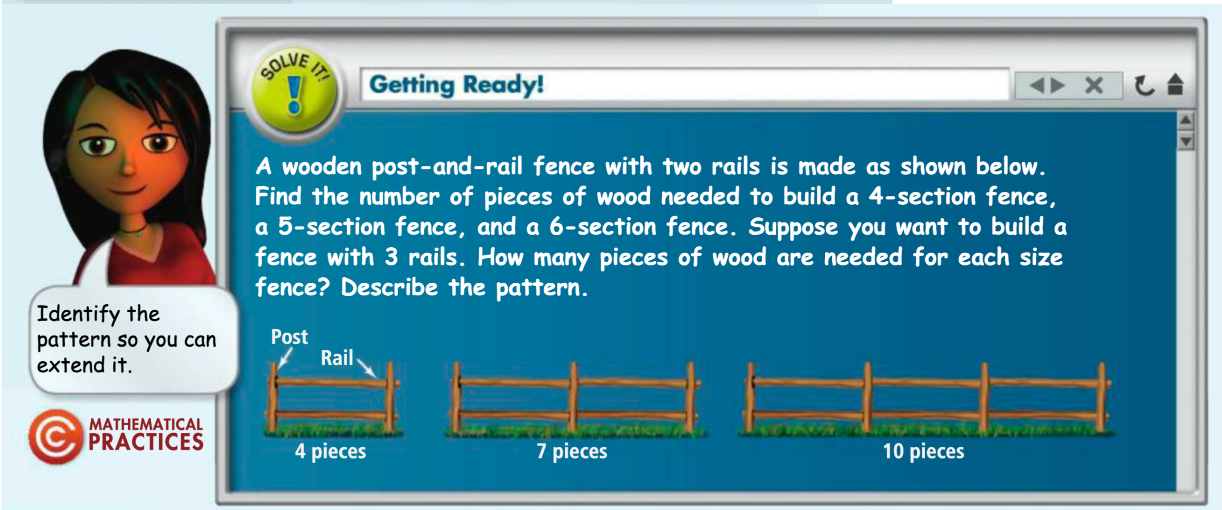
4-7 Arithmetic Sequences



In the Solve It, the numbers of pieces of wood used for 1 section of fence, 2 sections of fence, and so on, form a pattern, or a sequence. A ***sequence*** *is an ordered list of numbers that often form a pattern.* Each number in the list is called a ***term of a sequence***.

When you can identify a pattern in a sequence, you can use it to extend the sequence. You can also model some sequences with a function rule that you can use to find any term of sequence.

PROBLEM 1: EXTENDING SEQUENCES

Describe a pattern in each sequence. What are the next two terms?

a) 5, 8, 11, 14, …
+3 +3 +3
add 3 to previous term
17, 20

b) 2.5, 5, 10, 20, …
~~+2.5 +2.5~~
·2 ·2 ·2
multiply by two
40, 80

c) 5, 11, 17, 23, …
+6 +6 +6
add 6
29, 35

d) 2, -4, 8, -16, …
·(-2) ·(-2) ·(-2)
Multiply by -2
32, -64

e) 400, 200, 100, 50, …
÷ 2
$\cdot \frac{1}{2}$
25, 12.5

f) 1, 1, 2, 3, 5, 8, **13**, …
FIBONACCI SEQUENCE

In an ***arithmetic sequence***, *the difference between consecutive terms is constant. This difference is called the* ***common difference***.

PROBLEM 2: IDENTIFYING AN ARITHMETIC SEQUENCE

Tell whether the sequence is arithmetic. If it is, what is the common difference?

+5 +5 +5

a) 3, 8, 13, 18, …

Arithmetic

Common difference = +5

+3 +4 +4

b) 6, 9, 13, 17, …

Not arithmetic

-6 -6 -6

c) 10, 4, -2, -8, …

Arithmetic

common difference = -6

+7 +7 +8

d) 8, 15, 22, 30, …

Not arithmetic

-4 +4 -4

e) 2, -2, 2, -2, …

·(-1) ·(-1) ·(-1)

Not arithmetic

+4 +4 +4

f) -15, -11, -7, -3, …

Arithmetic

Common difference = +4

A sequence is a function whose domain is the natural numbers, and whose outputs are the terms of the sequence.

You can write a sequence using a recursive formula. A ***recursive formula*** *is a function rule that relates each term of a sequence after the first to the ones before it.*

$A(n) = A(n-1) + \text{common difference}$

PROBLEM 3: WRITING A RECURSIVE FORMULA

Write a recursive formula for the arithmetic sequences below. What is the value of the 8th term?

+4 +4 +4

a) 7, 11, 15, 19, …

$\boxed{A(n) = A(n-1) + 4}$

$A(1) = 7$

$A(2) = 11$

$A(3) = 15$

$A(4) = 19$

$A(5) = A(4) + 4 = 19 + 4 = 23$

$A(6) = A(5) + 4 = 23 + 4 = 27$

$A(7) = A(6) + 4 = 27 + 4 = 31$

$A(8) = A(7) + 4 = 31 + 4 = 35$

+7 +7 +7

b) 70, 77, 84, 91, …

$A(n) = A(n-1) + 7$

+12 +12 +12

c) 23, 35, 47, 59, …

$A(n) = A(n-1) + 12$

+6 +6 +6 +6 +6 +6 +6

d) 3, 9, 15, 21, …

$A(n) = A(n-1) + 6$

$A(n) = 3 + (n-1)(6)$

$A(8) = 3 + (7)(6) = 3 + 42 = 45$

+0.5 +0.5 +0.5

e) 7.3, 7.8, 8.3, 8.8, …

$A(n) = A(n-1) + 0.5$

-9 -9 -9

f) 97, 88, 79, 70, …

$A(n) = A(n-1) - 9$

You can find the value of any term of an arithmetic sequence using a recursive formula. You can also write a sequence using an explicit formula. An ***explicit formula*** *is a function rule that relates each term of a sequence to the term number*.

KEY CONCEPT: EXPLICIT FORMULA OF AN ARITHMETIC SEQUENCE

The *n*th term of an arithmetic sequence with the first term *A(1) and common difference d is given by:*

$$A(n) = A(1) + (n-1)d$$

nth term 1st term common difference

PROBLEM 4: WRITING AN EXPLICIT FORMULA

a) An online auction works as shown below. Write an explicit formula to represent the bids as an arithmetic sequence. What is the twelfth bid?

$A(n)=A(1)+(n-1)d$

$\boxed{A(n) = 200 + (n-1)(10)}$

$A(12)=200+(11)(10)$

$=200+110$

$=310$

\$310

b) A subway pass has a starting value of \$100. After one ride, the value of the pass is \$98.25. After two rides, the value is \$96.50. After three rides, its value is \$94.75. Write an explicit formula to represent the remaining value on the card as an arithmetic sequence. What is the value of the pass after 15 rides?

rides	$ left	
0	100	-1.75
1	98.25	-1.75
2	96.50	-1.75
3	94.75	

$A(n)=A(1)+(n-1)(d)$

$\boxed{A(n)=98.25+(n-1)(-1.75)}$

$A(15)=98.25+(14)(-1.75)$

$=73.75$

\$73.75

c) How many rides can be taken with the \$100 pass?

$A(n)=98.25+(n-1)(-1.75)$

$0=98.25+(n-1)(-1.75)$

$-98.25 \quad -98.25$

$\frac{-98.25}{-1.75}=\frac{(n-1)\cancel{(-1.75)}}{\cancel{-1.75}}$

$56.14 = n-1$
$+1 \quad +1$
$57.14 = n$

57 rides

d) After one customer buys 4 new tires, a garage recycling bin has 20 tires in it. After another customer buys 4 new tires, the bin has 24 tires in it. Write an explicit formula to represent the number of tires in the bin as an arithmetic sequence. How many tires are in the bin after 9 customers buy all new tires?

cust.	tires in bin	
1	20	
2	24	+4
3	28	+4

$A(n) = A(1) + (n-1)(d)$

$\boxed{A(n) = 20 + (n-1)(4)}$

$A(9) = 20 + (8)(4)$
$= 20 + 32$
$= 52$

52 tires

e) You have a gift card at the local donut shop worth \$50. After you buy an iced-coffee on Monday, its value is \$46.75. After you buy your iced-coffee on Tuesday, its value is \$43.50. Write an explicit formula to represent the amount of money left on the card as an arithmetic sequence. What is the value of the card after you buy 12 iced-coffees?

coffees	\$ left	
0	50	
1	46.75	−3.25
2	43.50	−3.25

$A(n) = A(1) + (n-1)(d)$

$\boxed{A(n) = 46.75 + (n-1)(-3.25)}$

$A(12) = 46.75 + (11)(-3.25)$
$= 11$

\$11

You can write an explicit formula from a recursive formula and vice versa.

PROBLEM 5: WRITING AN EXPLICIT FORMULA FROM A RECURSIVE FORMULA

Write an explicit formula for each recursive formula. The first term of each sequence is given.

a) $\boxed{A(n) = A(n-1) + 12;}\ A(1) = 19$

c.d.

$A(n) = 19 + (n-1)(12)$

b) $A(n) = A(n-1) + 7;\ A(1) = -2$

$A(n) = -2 + (n-1)(7)$

c) $A(n) = A(n-1) + 2;\ A(1) = 21$

$A(n) = 21 + (n-1)(2)$

d) $A(n) = A(n-1) - 0.3;\ A(1) = 0.3$

$A(n) = 0.3 + (n-1)(-0.3)$

PROBLEM 6: WRITING A RECURSIVE FORMULA FROM AN EXPLICIT FORMULA

Write a recursive formula from each given explicit formula.

a) $A(n) = 32 + (n - 1)(22)$

$A(n) = A(n-1) + 22$

b) $A(n) = 1 + (n - 1)(3)$

$A(n) = A(n-1) + 3$

c) $A(n) = 76 + (n - 1)(10)$

$A(n) = A(n-1) + 10$

d) $A(n) = -1 + (n - 1)(-2)$

$A(n) = A(n-1) - 2$

SUPERSTAR PROBLEMS:

a) Suppose the first Friday of a new year is the fourth day of that year. Will the year have 53 Fridays regardless of whether or not it is a leap year?

b) The first five rows of Pascal's Triangle are shown at the right.

i) Predict the numbers in the seventh row.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

ii) Find the sum of the numbers in each of the first five rows. Predict the sum of the numbers in the seventh row.

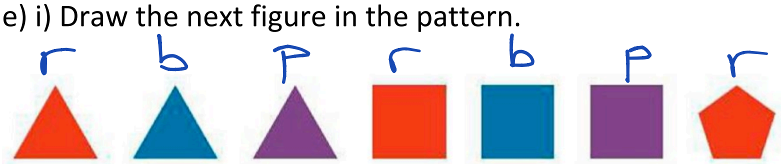
c) Buses run every 9 minutes starting at 6:00 a.m. You get to the bust stop at 7:16 a.m. How long will you wait for a bus?

d) Find the common difference of each arithmetic sequence. Then find the next term.

i) 4, $x + 4$, $2x + 4$, $3x + 4$, …

ii) $a + b + c$, $4a + 3b + c$, $7a + 5b + c$, …

e) i) Draw the next figure in the pattern.



ii) What is the color of the twentieth figure? Explain.

iii) How many sides does the twenty-third figure have? Explain.

Name ______________________ Class ____________ Date ____________

4-7 Homework

Describe the pattern in each sequence. Then find the next two terms of the sequence.

1. 3, 6, 12, 24, …

2. 9, 15, 21, 27, …

3. 1.5, 2.25, 3, 3.75, …

4. 9.9, 8.8, 7.7, 6.6, …

5. 1.5, 4.5, 13.5, 40.5, …

6. 40, 20, 10, 5, …

7. 7, 11, 15, 19, …

8. 67, 60, 53, 46, …

9. 12, 7, 2,-3, …

Tell whether the sequence is arithmetic. If it is, identify the common difference.

10. 4, 8, 12, 16, …

11. –11, 5, 0, 6, …

12. 4, 8, 16, 32, …

13. 12, 23, 34, 45, …

14. 2, 4, 7, 9, …

15. 1, 3, 9, 27, …

16. –16, –11, –6, –1, …

17. –9, –4.5, –0.5, 4, …

18. –7, –14, –21, –28, …

19. Th e pic …

20. 5, 10, 15, 20, …

21. 2, 20, 200, 2000, …

22. You have a gift card for a coffee shop worth $90. Each day you use the card to get a coffee for $4.10. Write a rule to represent the amount of money left on the card as an arithmetic sequence. What is the value of the card after buying 8 coffees?

23. You start a savings account with $200 and save $30 each month. Write a rule to represent the amount of money you invest into your savings account as an arithmetic sequence. How much money will you have invested after 12 months?

Find the third, fifth, and tenth terms of the sequence described by each rule.

24. $A(n) = 4 + (n + 1)(-5)$

25. $A(n) = 2 + (n + 1)(6)$

26. $A(n) = -5.5 + (n - 1)(2)$

27. $A(n) = 3 + (n - 1)(1.5)$

28. $A(n) = -2 + (n - 1)(5)$

29. $A(n) = 1.4 + (n - 1)(3)$

30. $A(n) = 9 + (n - 1)(8)$

31. $A(n) = 2.5 + (n - 1)(2.5)$

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

32. 1.6, 0.8, 0, –0.8, …

33. 5, 10, 20, 40, …

34. 5, 13, 21, 29, …

35. 51, 47, 43, 39, …

36. 0.2, 0.5, 0.8, 1.1, …

37. 7, 14, 28, 56, …

38. **Open-Ended** Write an explicit formula for the arithmetic sequence whose common difference is –2.5.

39. **Error Analysis** Your friend writes $A(8) = 3 + (8)(5)$ as an explicit formula for finding the eighth term of the arithmetic sequence 3, 8, 13, 18, … Describe and correct your friend's error.

40. The local traffic update is given on a radio channel every 12 minutes from 4:00 p.m. to 6:30 p.m. You turn the radio on at 4:16 p.m. How long will you wait for the local traffic update?

$$\begin{array}{c|l} 3 & 7, 11, 15, 19, \ldots \\ \hline 0 & \end{array}$$

$7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19$

50th term is 203

$A(50) = 203$

$$\boxed{A(n) = 4n + 3}$$

$$\begin{array}{c|l} -5 & 5, 15, 25, 35, \ldots \\ \hline 0 & \end{array}$$

50th term is 495

$A(50) = 495$

$A(n) = 10n - 5$